Tutorial 1 EM203

1. $F(x) = e^{-x}$ $\quad x_i = 0.25 \quad x_{i+1} = 1$

$$h = x_{i+1} - x_i$$
$$= 1 - 0.25$$
$$h = 0.75$$

0^{th} order

$$f(x_{i+1}) \simeq f(x_i)$$
$$e^{-1} \simeq e^{-0.25}$$
$$0.3679 \simeq 0.7788$$

$$|\varepsilon_t| = \left| \frac{\text{true value} - \text{approximate value}}{\text{true value}} \right| \times 100\%$$

$$|\varepsilon_t| = \left| \frac{0.3679 - 0.7788}{0.3679} \right| \times 100\%$$

$$|\varepsilon_t| = 111.688\%$$

1st order $\qquad f'(x) = -e^{-x}$

$$f(x_{i+1}) \simeq f(x_i) + f'(x_i)h$$

$$\simeq f(0.25) + f'(0.25)(0.75)$$

$$\simeq 0.7788 + (-0.7788 \times 0.75)$$

$$= 0.1947$$

$$|\varepsilon_t| = \left|\frac{0.3679 - 0.1947}{0.3679}\right| \times 100\%$$

$$= 47.078\%$$

2nd order. $\qquad f''(x) = e^{-x}$

$$f(x_{i+1}) \simeq f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!}$$

$$\simeq 0.7788 + (-0.7788 \times 0.75)$$

$$+ \frac{0.7788 \times (0.75)^2}{2}$$

$$\simeq 0.4137$$

$$|\varepsilon_t| = \left|\frac{0.3679 - 0.4137}{0.3679}\right| \times 100\%$$

$$= 12.449\%$$

3rd order $f'''(x) = -e^{-x}$

$$f(x_{i+1}) \simeq f(x_i) + f'(x_i)h + \frac{f''(x_i)h^2}{2!} + \frac{f'''(x_i)h^3}{3!}$$

$$\simeq 0.4137 + \left(\frac{-0.7788 \times (0.75)^3}{6}\right)$$

$$\simeq 0.3589$$

$$|\epsilon_t| = \left|\frac{0.3679 - 0.3589}{0.3679}\right| \times 100\%$$

$$\simeq 2.4463\%$$

2. $f(x) = 25x^3 - 6x^2 + 7x - 88$

$x_{i+1} = 3 \quad x_i = 1 \quad h = 3 - 1 = 2$

$$f(x_{i+1}) \simeq f(x_i)$$

$$f(3) = 554$$

$$\neq f(1) = -62$$

$$|\epsilon_t| = \left|\frac{554 - -62}{554}\right| \times 100\%$$

$$= 111.19\%$$

1^{st} order $\qquad f(x) = 75x^2 - 12x + 7$

$$F(x_{i+1}) \approx F(x_i) + F'(x_i)h$$

$$\simeq -62 + 70 \times 2$$

$$\simeq 78$$

$$|\varepsilon_t| = \left|\frac{554 - 78}{554}\right| \times 100\%$$

$$\cancel{98.5559} = 85.92\%$$

2^{nd} order $\qquad F''(x) = 150x - 12$

$$F(x_{i+1}) \simeq F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!}$$

$$\simeq 78 + 276$$

$$\simeq 354$$

$$|\varepsilon_t| = \left|\frac{554 - 354}{554}\right| \times 100\%$$

$$= 36.10\%$$

3^{rd} order $\qquad F'''(x) = 150$

$$F(x_{i+1}) \simeq F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!} + \frac{F'''(x_i)h^3}{3!}$$

$$f(x_{i+1}) \approx 354 + 150$$
$$\approx 504$$

$$|e_t| = \left|\frac{554 - 504}{554}\right| \times 100\%$$

$$= 9.025\%$$

3. $f(x) = \ln(x)$

$x_{i+1} = 3 \quad x_i = 1 \quad h = 2$

0^{th} order

$$f(x_{i+1}) \approx f(x_i)$$
$$1.0986 \approx 0$$

$$|e_t| = \left|\frac{1.0986 - 0}{1.0986}\right| \times 100\%$$

$$= 100\%$$

1^{st} Order $\qquad f'(x) = 1/x$

$$f(x_{i+1}) \approx f(x_i) + f'(x_i)\,h$$
$$\approx 0 + 1 \times 2$$
$$\approx 2$$

$$|e_t| = \left|\frac{1.0986 - 2}{1.0986}\right| \times 100\%$$

$$|e_t| = 82.0198\%$$

2nd order $\qquad F''(x) = -1/x^2$

$$F(x_{i+1}) \simeq F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!}$$

$$\simeq 2 + \left(\frac{-1 \times 4}{2}\right)$$

$$\simeq 0$$

$$|e_t| = 100\%$$

3rd order $\qquad F'''(x) = 2/x^3$

$$F(x_{i+1}) \simeq F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!} + \frac{F'''(x_i)h^3}{3!}$$

$$\simeq 0 + \frac{2 \times 2^3}{6}$$

$$\simeq 2.667$$

$$|e_t| = \left|\frac{1.0986 - 2.667}{1.0986}\right| \times 100\%$$

$$= 142.733\%$$

4th order $\quad F^{4}(x) = -6/x^4$

$$F(x_{i+1}) \approx F(x_i) + F'(x_i)h + \frac{F''(x_i)h^2}{2!}$$

$$+ \frac{F'''(x_i)h^3}{3!} + \frac{F^{4}(x_i)h^4}{4!}$$

$$\approx 2.667 + \frac{-6 \times 2^4}{4 \times 3 \times 2}$$

$$\approx -1.333$$

$$|\varepsilon_t| = \left| \frac{1.0986 + 1.333}{1.0986} \right| \times 100\%$$

$$= 221.33\%$$

error is ocilating and increase.

$$\text{Condition NO} = \frac{\bar{x} F'(\bar{x})}{F(\bar{x})}$$

$$= \frac{1 \times 1}{0}$$

$$= \infty$$

$\therefore$ Function is ill conditoin

4. $f(x) = \sin(\sqrt{x}) - x = 0$

$$x = \sin(\sqrt{x})$$
$$x = g(x)$$

$x_0 = 0.5$

$$x_1 = g(x_0) = \sin\sqrt{0.5}$$
$$= 0.6496$$
$$x_2 = g(x_1) = \sin\sqrt{0.6496}$$
$$= 0.72152$$

$$e_a = \left| \frac{\text{current value} - \text{Previous value}}{\text{current value}} \right| \times 100$$

x_i	e_a %
0.5	–
0.6496	23.0295
0.72152	9.9678
0.75089	3.91135
0.76209	1.4696
0.76625	0.5429
0.76777	0.1979
0.76833	0.07288
0.76853	0.0260
0.76860	0.009107

$\therefore$ root value ≈ 0.76860

In this process error decrease monotonically. so this process is linearly convergent. $(E_{t,i+1} \propto E_{t,i})$

$$\cancel{g'(\xi) = \frac{x_r - x_{i+1}}{x_r - x}}$$

5. $$f(x) = 7\sin(x)\, e^{-x} - 1$$

$$f'(x) = 7[\sin(x) \times -e^{-x} + e^{-x}\cos x]$$

$$f'(x) = 7[\cos x - \sin x]\, e^{-x}$$

a). $x_r = 0.17017999$

b) $$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$i=0$, $x_0 = 0.3$ $$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.3 - \frac{0.5325}{3.4216}$$

$$= 0.1444$$

$i=1, \quad x_1 = 0.1444 \qquad x_2 = x_1 - \dfrac{F(x_1)}{F'(x_1)}$

$$= 0.144 - \frac{(-0.1281)}{5.1239}$$

$$= 0.1694$$

$i=2; \quad x_2 = 0.1694 \qquad x_3 = 0.1694 - \dfrac{F(x_2)}{F'(x_2)}$

$$= 0.1694 - \frac{(-0.0038)}{4.8284}$$

$$= 0.17017$$

$i=3; \quad x_3 = 0.1702 \qquad x_4 = x_3 - \dfrac{F(x_3)}{F(x_3)}$

$$= 0.17017 - \frac{(9.6414 \times 10^{-5})}{4.8191}$$

$$= 0.1701799$$

$\therefore x_r = 0.1702799$

6. $f(x) = 0.0074x^4 - 0.284x^3 + 3.355x^2 - 12.183x + 5$

$$f'(x) = 0.0296x^3 - 0.852x^2 + 6.71x - 12.183$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$i=0 \quad x_0 = 16.15 \qquad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$= 16.15 - \frac{(-9.574454)}{(-10.07468)}$$

$$= 15.199$$

$i=1 \quad x_1 = 15.199 \qquad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$= 15.199 - \frac{(-7.384619)}{(-11.296348)}$$

$$= 14.545282$$

$i=2 \quad x_2 = 14.545 \qquad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$$= 14.545 - \frac{(-5.12627)}{(-11.604866)}$$

$$= 14.1032$$

$i=3 \quad x_3 = 14.1032 \quad x_4 = 14.1032 - \frac{(-3.41217)}{(-11.59742)}$

$= 13.80898$

x_{i+1} value is < value in between 15 - 20

so By this method we cannot get root value between 15 and 20.
By this method converge to root value under 15.

7. $e^x + x^2 = 100$

$e^x = 100 - x^2$

$x \ln(e) = \ln(100 - x^2)$

$x = \ln(100 - x^2)$

$x = g(x)$

$e^4 = 54.59$ $\quad$ ∴ Inial guess $x_0 = 4$

$e^5 = 148.41$

x_i	ε_a %
4	—
4.4308	9.7238
4.3866	1.0072
4.39145	0.1105
4.39092	0.012

$x_r \approx 4.39092$

error monotonic decrease.
so iterative scheme will converge definitely to the desired root.

8. $xe^x - 20 = 0$

$f(x) = xe^x - 20$

$f'(x) = xe^x - e^x$

$f'(x) = e^x(x-1)$

Initial value $= 2$ (By Graph)

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$i = 0 \quad x_0 = 2 \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$= 2 - \frac{(-5.2218)}{(7.38905)}$$

$$= 2.7067$$

$i = 1 \quad x_1 = 2.7067 \quad x_2 = 2.7067 - \frac{(20.54571)}{(25.56595)}$

$$= 1.903064$$

$i=2$ $x_2 = 1.903064$ $x_3 = 1.903064 - \frac{(-7.23726)}{(6.056318)}$

$= 3.09805$

$i=3$ $x_3 = 3.09805$ $x_4 = 3.09805 - \frac{(48.63634)}{(46.481683)}$

$= 2.05169$

$i=4$ $x_4 = 2.05169$ $x_5 = 2.05169 - \frac{(-4.035718)}{(8.18334)}$

$= 2.54485$

x_i :- 2.7067
1.903064
3.09805
2.05149
2.54485

x_i value is ocillating between 2. so this function NR method show poor convergenee.

9.a. $F(x) = x^3 - x^2 - x - 1$

$$x_{i+1} = x_i - \frac{F(x_i)}{F'(x_i)}$$

$$F'(x) = 3x^2 - 2x - 1$$

$i=0$ $x_0 = 2$ $x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$

$= 2 - \frac{1}{7}$

$= 1.8571428$

$i=1$ $x_1 = 1.85714$ $x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}$

$= 1.85714 - \frac{0.0991}{5.63262}$

$= 1.839546$

$i=2$ $x_2 = 1.8395$ $x_3 = x_2 - \frac{F(x_2)}{F'(x_2)}$

$= 1.8395 - \frac{0.001167}{5.472280}$

$= 1.8392867$

$i=3$ $x_3 = 1.83929$ $x_4 = x_3 - \frac{F(x_3)}{F'(x_3)}$

$= 1.83929 - \frac{1.7750 \times 10^{-5}}{5.47038}$

$$x_4 = 1.839286$$

$\therefore x_r \approx 1.8392$ [Four decimal places]

b. $x = x^3 - x^2 - 1$

$x = g(x) \qquad g'(x) = 3x^2 - 2x$

~~$x_0 = 2 \qquad x_1 = g(x_0) =$~~

$f(x) = x \qquad g(x) = x^3 - x^2 - 1$

$f'(x) = 1 \qquad g'(x) = 3x^2 - 2x$

$x = 2 \qquad f'(2) = 1 \qquad g'(2) = 3 \times 4 - 2 \times 2$

$= 8$

$1 < 8$

$f'(2) < g'(2)$

$|g'(2)| > 1$

$\therefore$ Divergence

$x = (x^2 + x + 1)^{1/3}$

$x = g(x) \qquad g'(x) = \frac{1}{3}(x^2 + x + 1)^{-2/3} \times [2x + 1]$

$f(x) = x \qquad f'(x) = 1$

$x = 2 \qquad g'(2) = \frac{1}{3}(4 + 2 + 1)^{-2/3} \times (4 + 1)$

$= 0.455$

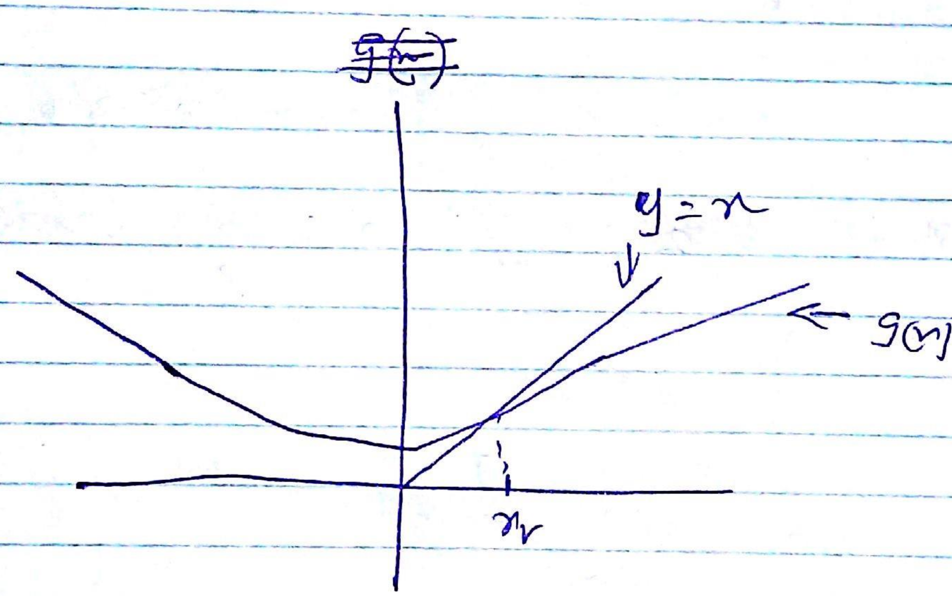
(17) 3)

$$|g'(2)| < 1$$

$\therefore$ Convergence.

$\therefore$ so $x = (x^2+x+1)^{1/3}$ equation can be use to find roots by fixed itteration method.

By we can get only one root.
Because $g(x) = (x^2+x+1)^{1/3}$ graph



so there is only one intersection point. so we can get only one root.

ii $x_0 \doteq 2$ $\quad x_1 = g(x_0) = (x^2+x+1)^{1/3}$

x	$\varepsilon_a\%$
2	—
1.9129311	4.55159
1.8731435	2.124108
1.8548749	0.984898
1.846486	0.455626339
1.84259667	0.21013687
1.8408124	0.096927714
1.839990	0.044694
1.839610	0.02060653
1.839436	0.0094999
1.839355	0.0043795
1.8393185	0.00201859542
1.83930139	9.3072×10^{-4}
1.8392935	4.2905864×10^{-4}
1.839289	$1.977928066\times10^{-4} \Rightarrow 0.0001$

$\therefore\ x_r = 1.8392$ [four decimal places]